

Giant Optical Non-linearity induced by a Single Two-Level System interacting with a Cavity in the Purcell Regime

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Abstract

A two-level system that is coupled to a high-finesse cavity in the Purcell regime exhibits a giant optical non-linearity due to the saturation of the two-level system at very low intensities, of the order of one photon per lifetime. We perform a detailed analysis of this effect, taking into account the most important practical imperfections. Our conclusion is that an experimental demonstration of the giant non-linearity should be feasible using semiconductor micropillar cavities containing a single quantum dot in resonance with the cavity mode. We also discuss the perspectives for practical applications of the effect.

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I. INTRODUCTION

The implementation of giant optical non-linearities is of interest both from the fundamental point of view of realizing strong photon-photon interactions, and because it is hoped that such an implementation would lead to applications in classical and quantum information processing. One particularly promising system for realizing large non-linearities is a single emitter (two-level system) embedded in a high-finesse cavity, which serves to enhance the interaction between the emitter and the electro-magnetic field. In the so-called strong coupling regime, where the interaction between the emitter and the light dominates over all other processes including cavity decay, there are well-known dramatic non-linear effects such as normal-mode splitting [1], vacuum Rabi oscillations [2] and photon blockade [3].

While there have been several pioneering experiments both for free-space cavities containing single atoms [1, 2, 3] and for semiconductor microcavities containing single quantum dots [4], the conditions for strong coupling are quite challenging. It is therefore of interest to consider the potential for large optical non-linearities in the so-called Purcell regime, where the interaction between the emitter and the cavity mode dominates over that with all other modes, but where the cavity decay is still faster than the emitter lifetime. The Purcell regime is significantly easier to attain in practice than strong coupling, in particular it has also been reached for single-photon sources based on micropillars containing quantum dots [5, 6, 7].

It is in fact possible to have giant optical non-linearities in the Purcell regime [8, 9, 10]. In the present work we study the very simplest non-linear effects, namely those related to the saturation of a single two-level system by light that is in, or close to resonance with the two-level transition. In Ref. [11] it was described how the presence of a single resonant dipole can dramatically change the absorption/transmission properties of a high-finesse cavity, and it was proposed to control the coupling between the light and the dipole by exploiting additional levels of the system (e.g. for a charged quantum dot). In the present situation, the coupling between the light and the dipole is governed by the intensity of the light itself. When the intensity is sufficiently high, the dipole becomes saturated and thus effectively decouples from the light. Since the saturation occurs at intensity levels of order one photon per lifetime of the emitter, this effectively realizes a strong interaction between individual photons, that is to say, a giant optical non-linearity.

A pioneering experiment on optical non-linearities in the Purcell regime was performed with atoms in a free-space cavity in a slightly off-resonant configuration [8]. To our knowledge, the first theoretical study was performed in Ref. [10], based on the “one-dimensional atom” model suggested in Ref. [12], for the case of a one-sided cavity and for exact resonance between the light and the emitter. In the present work, in section II, we establish the coupled-mode equations for the cavity mode and for the input and output fields following Ref. [13]. This allows us to generalize the results of Ref. [10] to non-resonant situations and to double-sided cavities. In section III the stationary solution of these equations is derived in the linear (low-intensity) case: we show that the two-level system induces a dip in the transmission of the system. We derive the expression for the transmission of the optical system in the presence of losses. In section IV we treat the case of general intensities via a semi-classical approximation [10], which allows to show the giant optical non-linearity. An important motivation for our work is to study the feasibility of an experimental demonstration of the non-linearity with a semiconductor micropillar cavity containing a single quantum dot. In section V we give detailed quantitative estimates for this system and show that the non-linear effect should be observable using state-of-the-art microcavities. In section VI we discuss the perspectives for applications of the non-linearity in the context of classical or quantum information processing.

II. QUANTUM COUPLED-MODE EQUATIONS

The situation considered is represented on figure 1. A single mode of the electromagnetic field is coupled to the outside world via two ports labelled 1 et 2. Each port supports a one-dimensional continuum of modes respectively labelled by the subscripts k and l . This may correspond to the case of a high finesse Fabry-Perot made of two partially reflecting mirrors. Among the infinity of modes supported by the cavity, we consider only one mode that interacts with two continua of planewaves through the left and the right mirror. The cavity contains a single two-level system of frequency ω_0 which is nearly on resonance with the mode of interest. We note a , b_k , c_l the annihilation operator for the cavity mode, the modes of port 1 and port 2 respectively, $\omega_0 + \delta$, ω_k and ω_l the corresponding frequencies. The atomic operators are $S_z = \frac{1}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$ and $S_- = |g\rangle\langle e|$. The coupling strengths between the cavity and the modes of port 1 and 2 are taken constant and equal to g_1 and

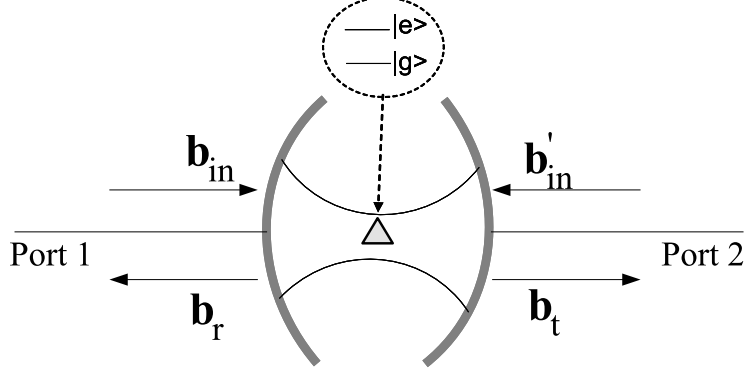


FIG. 1: Scheme of the atom-cavity coupled system. The atomic frequency is ω_0 , the cavity mode frequency $\omega_0 + \delta$. The cavity mode is coupled to the outside world via two ports labelled 1 and 2 modes with coupling constants g_1 and g_2 , the two-level system to the cavity mode with coupling constant Ω . The situation can describe a micropillar containing a single quantum dot.

g_2 respectively. The total Hamiltonian of the system is then

$$H = \hbar\omega_0 S_z + \hbar(\omega_0 + \delta)a^\dagger a + \sum_k \hbar\omega_k b_k^\dagger b_k + \sum_l \hbar\omega_l c_l^\dagger c_l + i\hbar\Omega(S_+ a - a^\dagger S_-) + i\hbar \sum_k (g_1 a^\dagger b_k - g_1^* b_k^\dagger a) + i\hbar \sum_l (g_2 a^\dagger c_l - g_2^* c_l^\dagger a). \quad (1)$$

The first four terms represent the free evolution of the atom, the cavity field, the modes in port 1 and 2 respectively. The last three terms represent the atom-cavity coupling, the coupling of the cavity mode with the modes of port 1 and with the modes of port 2. We can write the Heisenberg equations for each operator

$$\begin{aligned} \dot{S}_- &= -i\omega_0 S_- - 2\Omega S_z a \\ \dot{S}_z &= \Omega(S_+ a + a^\dagger S_-) \\ \dot{a} &= -i(\omega_0 + \delta)a - \Omega S_- + g_1 \sum_k b_k + g_2 \sum_l c_l \\ \dot{b}_k &= -i\omega_k b_k - g_1^* a \\ \dot{c}_l &= -i\omega_l c_l - g_2^* a. \end{aligned} \quad (2)$$

We find for $b_k(t)$ and $c_l(t)$, for $t > t_0$ where t_0 is a reference of time

$$\begin{aligned} b_k(t) &= b_k(t_0)e^{-i\omega_k(t-t_0)} - g_1^* \int_{t_0}^t du a(u)e^{-i\omega_k(t-u)} \\ c_l(t) &= c_l(t_0)e^{-i\omega_l(t-t_0)} - g_2^* \int_{t_0}^t du a(u)e^{-i\omega_k(t-u)}. \end{aligned} \quad (3)$$

Equations (3) are then injected in the evolution equation for the cavity mode. For each mode b_k and c_l , the last term describes the field radiated by the cavity ("sources field")

and is responsible for the cavity damping. The first term describes the free evolution. It is responsible for the noise in the quantum Langevin equation. Here we don't trace over the modes k and l , but following Gardiner and Collett [13], we define the input field in each port

$$\begin{aligned} b_{in}(t) &= \frac{1}{\sqrt{\tau}} \sum_k b_k(t_0) e^{-i\omega_k(t-t_0)} \\ b'_{in}(t) &= \frac{1}{\sqrt{\tau}} \sum_l c_l(t_0) e^{-i\omega_l(t-t_0)}, \end{aligned} \quad (4)$$

where τ is defined by

$$\sum_k e^{-i\omega_k t} = \delta(t)\tau. \quad (5)$$

The quantity τ has the dimension of a time and depends on the mode density, which is supposed to be the same in each port. The quantity $b_{in}^+ b_{in}(t)$ (resp $b'^+_{in} b'_{in}(t)$) scales like a photon number per unit of time and represents the incoming power in port 1 (resp 2). Summing equations (3) over all modes in each port we have

$$\begin{aligned} \sum_k b_k(t) &= \sqrt{\tau} b_{in}(t) - \frac{g_1^*}{2} \tau a(t) \\ \sum_l c_l(t) &= \sqrt{\tau} b'_{in}(t) - \frac{g_2^*}{2} \tau a(t). \end{aligned} \quad (6)$$

We suppose for simplicity that the coupling to each port has the same intensity, $|g_1| = |g_2|$ and we write $g_i = |g_1| e^{i\theta_i}$, θ_1 and θ_2 being real numbers. This corresponds to the case of a symmetric Fabry-Perot cavity. The evolution equation for a becomes

$$\dot{a} = -i(\omega_0 + \delta)a - \kappa a - \Omega S_- + e^{i\theta_1} \sqrt{\kappa} b_{in} + e^{i\theta_2} \sqrt{\kappa} b'_{in}, \quad (7)$$

where $\kappa = |g_1|^2 \tau$ is the cavity linewidth. In the same way we can define the reflected and transmitted fields, for $t < t_0$

$$\begin{aligned} b_r(t) &= \frac{1}{\sqrt{\tau}} \sum_k b_k(t_0) e^{-i\omega_k(t-t_0)} \\ b_t(t) &= \frac{1}{\sqrt{\tau}} \sum_l c_l(t_0) e^{-i\omega_l(t-t_0)}, \end{aligned} \quad (8)$$

and in the same way we obtain

$$\begin{aligned} \sum_k b_k(t) &= \sqrt{\tau} b_r(t) + \frac{g_1^*}{2} \tau a(t) \\ \sum_l c_l(t) &= \sqrt{\tau} b_t(t) + \frac{g_2^*}{2} \tau a(t). \end{aligned} \quad (9)$$

From equations (6) and (9) we can easily derive the input-output equations for the two-ports cavity

$$\begin{aligned} b_r(t) &= b_{in}(t) - e^{-i\theta_1} \sqrt{\kappa} a \\ b_t(t) &= b'_{in}(t) - e^{-i\theta_2} \sqrt{\kappa} a. \end{aligned} \tag{10}$$



FIG. 2: Scheme of a cavity coupled quantum-dot system where the cavity is evanescently coupled to ports 1 and 2. The incoming field in port 1 is now entirely transmitted if the coupling with the cavity is switched off. This situation can describe a microdisk cavity evanescently coupled to a waveguide.

Note that this choice of definitions for the reflected and transmitted field depends on the geometry of the problem. In the situation depicted on figure 1, the incoming field in port 1 is entirely reflected if the coupling with the cavity is switched off. In the case of a cavity evanescently coupled to ports 1 and 2 (see figure 2) the incoming field in port 1 would be entirely transmitted if the coupling with the cavity were switched off. The definitions of b_r and b_t should just be inverted to describe this new situation. The Heisenberg equations for the cavity mode and the atomic operators are finally written in the frame rotating at the drive frequency ω

$$\begin{aligned} \dot{S}_- &= -i\Delta\omega S_- - 2\Omega S_z a \\ \dot{S}_z &= \Omega(S_+ a + a^\dagger S_-) \\ \dot{a} &= -i(\Delta\omega + \delta)a - \kappa a - \Omega S_- + e^{i\theta_1} \sqrt{\kappa} b_{in} + e^{i\theta_2} \sqrt{\kappa} b'_{in} \\ b_r &= b_{in} - e^{-i\theta_1} \sqrt{\kappa} a \\ b_t &= b'_{in} - e^{-i\theta_2} \sqrt{\kappa} a. \end{aligned} \tag{11}$$

Here $\Delta\omega = \omega_0 - \omega$. These equations are the quantum coupled-mode equations for the evolution of the atom and the cavity, driven by the external fields b_{in} and b'_{in} . At this stage we shall suppose that the cavity exchanges energy much faster with the input/output ports than with the atom, that is : $\kappa \gg \Omega$. This regime is often called the *bad cavity regime* and we will from now on restrict ourselves to that case. Note that the opposite case ($\Omega \gg \kappa$) corresponds to the *strong coupling regime* in which the emission of a photon by the atom is coherent and reversible, giving rise to the well-known phenomenon of quantum Rabi oscillation [2].

In the bad cavity regime, for a fixed frequency of the driving field, the cavity mode can be adiabatically eliminated from the equations, which means that we can take $\dot{a} = 0$ at each time of the system evolution. This implies for operator a

$$a = \frac{-\Omega S_- + \sqrt{\kappa}(e^{i\theta_1}b_{in} + e^{i\theta_2}b'_{in})}{i(\Delta\omega + \delta) + \kappa}. \quad (12)$$

The set of equations (11) becomes then

$$\begin{aligned} \dot{S}_- &= -i\Delta\omega S_- - \frac{\Gamma}{2}t_0(\Delta\omega)S_- + \sqrt{\frac{\Gamma}{2}}(-2S_z)(e^{i\theta_1}b_{in} + e^{i\theta_2}b'_{in})t_0(\Delta\omega) \\ \dot{S}_z &= -\Gamma\Re(t_0(\Delta\omega))\left(S_z + \frac{1}{2}\right) + \sqrt{\frac{\Gamma}{2}}\left(S_+(e^{i\theta_1}b_{in} + e^{i\theta_2}b'_{in})t_0(\Delta\omega) + hc\right) \\ b_t &= b'_{in}(1 - t_0(\Delta\omega)) - e^{i(\theta_1 - \theta_2)}b_{in}t_0(\Delta\omega) + e^{-i\theta_2}\sqrt{\frac{\Gamma}{2}}S_-t_0(\Delta\omega) \\ b_r &= b_{in}(1 - t_0(\Delta\omega)) - e^{i(\theta_2 - \theta_1)}b'_{in}t_0(\Delta\omega) + e^{-i\theta_1}\sqrt{\frac{\Gamma}{2}}S_-t_0(\Delta\omega). \end{aligned} \quad (13)$$

We have introduced the relaxation time of the dipole in the cavity mode $\Gamma = 2\Omega^2/\kappa$. We have denoted $t_0(\Delta\omega)$ the quantity $1/(1 + i(\Delta\omega + \delta)/\kappa)$. It will be shown in the next section that $-t_0$ corresponds to the transmission of an empty cavity. Note that equations (13) hold between operators. They are quantum equivalents for the well-known optical Bloch equations describing the dynamics of a two-level system interacting with a classical field. In the following we will be interested in the stationary solutions of the problem. We will eliminate S_- from the relations between the incoming and outgoing fields. Two specific cases emerge:

1. the incoming field is very weak, so that we can neglect the saturation term in the optical Bloch equations. As we shall see in the following section, we obtain linear equations

between incoming and outgoing field *operators*. The statistics of the incoming field is preserved by the transformation induced by the atom.

2. the incoming field has any intensity. Following Allen and Eberly [14], we shall adopt the semi-classical hypothesis where quantum correlations between the atom and the field can be neglected. As we will see in section IV this hypothesis allows us to derive the well-known optical Bloch equations, the source term due to the dipole fluorescence being entirely directed in the outgoing field.

III. LINEAR CASE

If the incoming field is very weak, the saturation of the two-level system can be neglected : the atomic population remains in the state $|g\rangle$, and we can replace S_z by its mean value $\langle S_z \rangle \approx -1/2$. Another way of introducing this approximation consists in noting that the behavior of a two-level system in a field containing very few excitations (zero or one photon) can't be distinguished from the behavior of a cavity because high energy levels are never populated. S_+ and S_- , which are analogous to creation and annihilation operators, should then have bosonic commutation relation. Given that $[S_-, S_+] = -2S_z$, this condition is fulfilled if $S_z \approx -1/2$. In a first part we shall focus on the case where the leaks can be neglected. In a second part, we will generalize the study to the case where the atom and the cavity can exchange energy with leaky modes.

A. Ideal case : the "one-dimensional atom"

In this case the atom can only desexcite itself in the cavity mode, and the cavity mode interacts only with two one-dimensional continua via two ports labelled 1 and 2. In particular spontaneous emission via the side of the cavity is assumed vanishingly small. This situation is generally referred to as the "one-dimensional atom" [12]. It is shown in appendix A that b_r and b_t are related to b_{in} and b'_{in} up to a global phase by a unitary transformation. Besides, it will be shown in appendix B that θ_1 and θ_2 depend on the choice of phase reference for the atomic dipole and for the field. Usually b_{in} and b'_{in} are taken real, and $\theta_1 = \theta_2 = \pi/2$.

We shall keep these values in the following. The scattering matrix \mathcal{S} reads then

$$\begin{pmatrix} b_r \\ b_t \end{pmatrix} = \mathcal{S} \begin{pmatrix} b_{in} \\ b'_{in} \end{pmatrix} = \frac{1}{1+i\zeta} \begin{pmatrix} i\zeta & -1 \\ -1 & i\zeta \end{pmatrix} \begin{pmatrix} b_{in} \\ b'_{in} \end{pmatrix}, \quad (14)$$

with

$$\zeta = \frac{\Delta\omega + \delta}{\kappa} - \frac{\Gamma}{2\Delta\omega}. \quad (15)$$

The system acts like a beamsplitter whose coefficients depend on the frequency of the incoming fields. The statistics is preserved by this transformation. If there is one photon of frequency ω in the input field, the output field will be a coherent superposition of a transmitted and a reflected photon of frequency ω , the amplitude of each part of the superposition corresponding to the coefficients of the diffusion matrix (14) as studied by Fan [15]. If the incoming field is quasi-classical, the outgoing field will be quasi-classical too and the reflection and transmission coefficients can be interpreted in the usual way. We consider the transmission coefficient in amplitude $t(\Delta\omega) = \mathcal{S}_{12} = \mathcal{S}_{21}$ which reads

$$t(\Delta\omega) = \frac{-1}{1+i\zeta}. \quad (16)$$

As mentioned previously, the transmission of the empty cavity, corresponding to $\Gamma = 0$, fulfills

$$t(\Delta\omega) = \frac{-1}{1+i\frac{\Delta\omega + \delta}{\kappa}} = -t_0(\Delta\omega). \quad (17)$$

The transmission coefficients in energy $T(\Delta\omega) = |t(\Delta\omega)|^2$ and $T_0(\Delta\omega) = |t_0(\Delta\omega)|^2$ are represented on figure 3 as functions of the normalized detuning between the cavity and the driving field $(\Delta\omega + \delta)/\kappa$. In all the article, we shall take $\Gamma = \kappa/500$ which is given by realistic experimental parameters. Note that this parameter corresponds to the bad cavity regime as it is defined in section II. If there is no atom in the cavity, $T_0(0) = 1$ and the field is entirely transmitted at resonance. If there is one resonant atom in the cavity, $T(0) = 0$ and the field is totally reflected by the optical system which behaves as a frequency selective perfect mirror as evidenced by Fan [15]. This effect can't be attributed to a phase-shift induced by

the atom, putting the cavity out of resonance. This can be understood by considering the expression of the transmitted field on resonance :

$$b_t = - \left[b_{in} + i\sqrt{\frac{\Gamma}{2}} S_- \right], \quad (18)$$

where the stationary state of the atomic dipole checks

$$S_- = i\sqrt{\frac{2}{\Gamma}} b_{in}. \quad (19)$$

The global $-$ sign in equation (18) is due to the cavity resonance. It appears that the field radiated by the dipole interferes destructively with the driving field and prevents light from entering the cavity. This destructive interference is due to the fact that the fluorescence field emitted by a two-level system is phase-shifted by π with respect to the driving field as pointed out by Kojima [16]. This phenomenon is reminiscent of the well-known dipole induced transparency as underlined by Waks et al. [11]. The two effects are similar if the cavity is evanescently coupled to a waveguide as presented on figure 2. In this case indeed, the light is blocked by the empty cavity and is transmitted if the cavity contains a dipole on resonance. If the dipole is not resonant with the cavity the transmission is a Fano resonance as underlined by Fan [15].

If $\delta = 0$, T reads

$$T(\Delta\omega) = \frac{1}{1 + \left(\frac{\Gamma}{2\Delta\omega} - \frac{\Delta\omega}{\kappa} \right)^2}. \quad (20)$$

The dip linewidths can be easily computed from the solutions of the equation $T = 1/2$. Remembering that $\Gamma \ll \kappa$, we find that the linewidth of the broadest transmission peak is the cavity linewidth

$$\Delta\omega_{1/2} = \kappa, \quad (21)$$

whereas the linewidth of the narrow dip corresponds to the linewidth of the atom dressed by the cavity mode

$$\delta\omega_{1/2} = \Gamma. \quad (22)$$

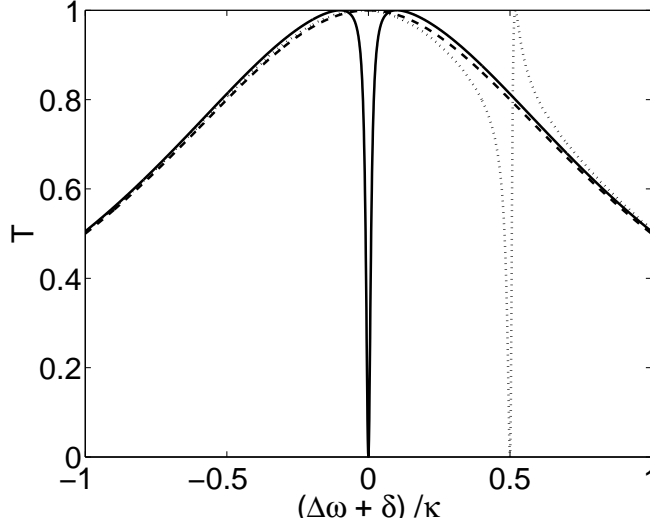


FIG. 3: Transmission of the optical system as a function of the normalized detuning $(\Delta\omega + \delta)/\kappa$ between the cavity and the driving frequency. The curves are plotted with $\Gamma/\kappa = 1/500$. Dashed : transmission of the empty cavity. Solid : transmission of the coupled quantum dot-cavity system, total reflection is induced by the dipole. Dots : transmission of the coupled quantum dot-cavity system with $\delta = -0.5\kappa$, the signal is typical for a Fano resonance.

B. Influence of the leaks

Until now we have studied the ideal case in which the atom is only coupled to the cavity mode and the cavity mode is only coupled to the atom and to two continua via the two ports 1 and 2. To evaluate what we can expect from a realistic experimental signal, we must take into account the unavoidable leaks from the cavity and the atom due to the couplings with undesired modes. We note γ_{at} and γ_{cav} the leaks from the atom and from the cavity respectively. Given that we will deal with artificial atoms such as quantum dots, we shall also consider the excitonic dephasing γ^* . The set of equations (11) becomes

$$\begin{aligned}
 \dot{S}_- &= -i\Delta\omega S_- - 2\Omega S_z a - \frac{\gamma_{at}}{2} S_- - \gamma^* S_- + G \\
 \dot{S}_z &= \Omega(S_+ a + a^+ S_-) - \gamma_{at}(S_z + 1/2) + K \\
 \dot{a} &= -i(\Delta\omega + \delta)a - \kappa a - \Omega S_- + i\sqrt{\kappa}b_{in} + i\sqrt{\kappa}b'_{in} - \frac{\gamma_{cav}}{2}a + H \\
 b_t &= b'_{in} + i\sqrt{\kappa}a \\
 b_r &= b_{in} + i\sqrt{\kappa}a .
 \end{aligned} \tag{23}$$

K , G and H are noise operators due to the interaction of the atom and the cavity with their respective reservoirs, respecting $\langle G \rangle = \langle H \rangle = \langle K \rangle = 0$. The noise prevents us from obtaining relations between incoming and outgoing field operators. As a consequence, we shall deal with expectation values of the fields as they could be obtained in a homodyne detection experiment, and denote $b_{in} = \langle b_{in} \rangle$, $b_{out} = \langle b_{out} \rangle$. As before we consider the linear case, so that $\langle S_z \rangle = -1/2$ and we denote $s = \langle S_- \rangle$. Moreover, we suppose $\langle b'_{in} \rangle = 0$. We obtain after adiabatic elimination of the cavity mode

$$\begin{aligned} \dot{s} &= -i\Delta\omega s - \frac{\Gamma}{2} \frac{Q}{Q_0} \left[t'_0 + \frac{Q_0}{Q} \frac{\gamma_{at} + 2\gamma^*}{\Gamma} \right] s + i \frac{Q}{Q_0} \sqrt{\frac{\Gamma}{2}} b_{in} t'_0 \\ b_t &= -\frac{Q}{Q_0} t'_0 b_{in} - i \frac{Q}{Q_0} \sqrt{\frac{\Gamma}{2}} t'_0 s \\ b_r &= b_{in} + b_t . \end{aligned} \tag{24}$$

We have introduced the adimensional quantity t'_0 such as

$$t'_0(\Delta\omega) = \frac{1}{1 + i \frac{Q}{Q_0} \frac{\Delta\omega + \delta}{\kappa}} . \tag{25}$$

The parameter Q_0 is the quality factor of the cavity mode due to the coupling with the one-dimensional continua of modes. The parameter Q is the total quality factor and includes the coupling to leaky ones. Q_0 and Q fulfill

$$Q_0/Q = 1 + \gamma_{cav}/2\kappa . \tag{26}$$

If the dipole is non-leaky, that is if $\gamma_{at} = 0$, its relaxation rate in the cavity mode is now equal to $\Gamma Q/Q_0$. It is lower than in the case of a cavity perfectly matched to the input and output modes, because the cavity being enlarged, the density of modes on resonance with the dipole is lower. It is convenient to define the ratio f

$$f = \frac{Q}{Q_0} \frac{\Gamma}{\gamma_{at} + 2\gamma^*} . \tag{27}$$

f is related to the Purcell factor F_p [18] of the two-level system by the following equation

$$f = \frac{\gamma_{free}}{\gamma_{at} + 2\gamma^*} F_p, \tag{28}$$

where γ_{free} is the dipole relaxation rate in free space. We define a transmission coefficient in amplitude $t = b_{out}/b_{in}$. The transmission coefficient of the empty cavity can be written

$-Q/Q_0 t'_0(\Delta\omega)$. If the cavity contains one atom, the transmission coefficient of the system reads now

$$t(\Delta\omega) = \frac{Q}{Q_0} t'_0 \left[-1 + \frac{f}{f + \left(\frac{i\Delta\omega}{\gamma_{at} + 2\gamma^*} + 1 \right) \left(i \frac{Q}{Q_0} \frac{\Delta\omega + \delta}{\kappa} + 1 \right)} \right], \quad (29)$$

At resonance, the transmission coefficient in energy for an empty cavity can be written

$$T_{max} = \left| \frac{Q}{Q_0} t'_0 \right|^2 = \left(\frac{Q}{Q_0} \right)^2. \quad (30)$$

To recover the ideal result $T_{max} = 1$ we obviously should have $Q = Q_0$. If the cavity contains one resonant two-level system, the transmission coefficient becomes

$$T_{min} = \left(\frac{Q}{Q_0} \right)^2 \left(\frac{1}{1+f} \right)^2. \quad (31)$$

The transmitted intensity decreases as f increases, that is to say as the two-level system is better coupled to the cavity mode. To recover the totally destructive interference obtained in the ideal case, we should have $f, F_p \rightarrow \infty$. This justifies for the so-called "Purcell regime" we have referred to until now. We will see in section V that Q and f are related, and we shall optimize the experimental parameters of a micropillar type microcavity to obtain a maximally contrasted signal $\mathcal{C} = T_{max} - T_{min}$.

By sake of completeness we have also analyzed the reflective properties of the system. As it is obvious on equation (24), the reflection coefficient in amplitude r checks $r = 1 + t$, on resonance we obtain for $R = |r|^2$ the following expression

$$R = \left(1 - \frac{Q}{Q_0} \frac{1}{1+f} \right)^2 \quad (32)$$

The system behaves as a perfect reflector in two quite different situations :

- $Q \ll Q_0$: the cavity is very bad, so that light cannot enter into it. We shall discard this case in the following.
- $Q/Q_0 \leq 1$ and $f \gg 1$: the cavity has some losses but contains an atom perfectly connected to the mode of interest. Let's point out the fact that the reflection may reach 1, even if the cavity is not perfect. Again this effect is due to a totally constructive

interference between the driving field and the field radiated by the optical system. Note that this feature has been pointed out by Fan [15], who has shown that a one-dimensional atom placed in a Fabry-Perot cavity is a perfect mirror at resonance.

We have plotted in figure 4 the evolution of T and R as functions of the atom-cavity detuning for different values of Q , Q_0 and f . On the plots (a) and (b), we consider the case of a cavity perfectly connected to the input and output mode ($Q = Q_0$) interacting with a leaky two-level system. On the plots (c) and (d), we consider the case of an atom perfectly connected to a leaky cavity mode ($f \rightarrow \infty$ and $Q/Q_0 < 1$). As expected in this case, the reflection is total.

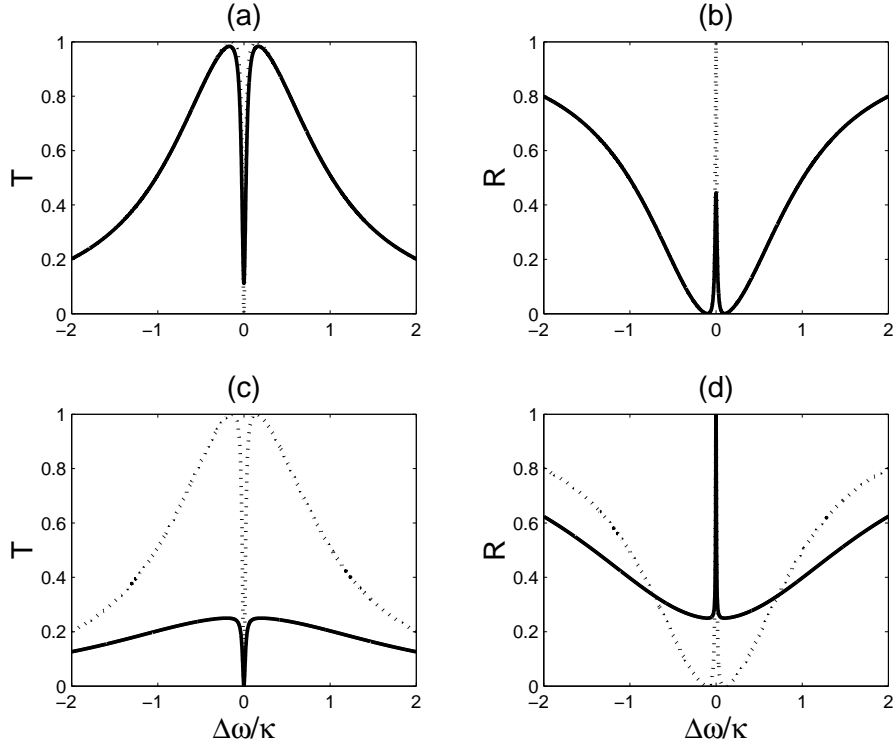


FIG. 4: *Evolution of T and R as functions of the atom-cavity detuning for different values of Q , Q_0 and f . δ has been taken equal to 0 for convenience. Dots : ideal case with $Q = Q_0$ and $f \rightarrow \infty$. (a) : T with $Q = Q_0$ and $f = 2$. (b) : R with the same parameters. (c) : T with $Q/Q_0 = 1/2$ and $f \rightarrow \infty$. (d) : R with the same parameters.*

It is convenient to compute the leaks on resonance \mathcal{L} given by $R + T = 1 - \mathcal{L}$. We easily

obtain

$$\mathcal{L} = 2\sqrt{R}\sqrt{T} = \frac{2Q}{Q_0} \frac{1}{1+f} \left(1 - \frac{Q}{Q_0} \frac{1}{1+f} \right) \quad (33)$$

We have $\mathcal{L} = 0$ for $R = 0$ (total transmission) or $T = 0$ (total reflection). The total transmission case can only be obtained with a perfect empty cavity ($Q = Q_0$ and $f = 0$). As it has been considered before, the total reflection case can be obtained with a leaky cavity containing an atom perfectly connected to its mode. This apparently striking result can be understood by noticing that \mathcal{L} can be approximated by the following expression in the case where $f \gg 1$

$$\mathcal{L} \sim \frac{2Q}{Q_0 f} = \frac{2\gamma_{at}}{\Gamma}, \quad (34)$$

which is the rate of photons lost by the atom over the rate of photons funneled in the output mode. As a consequence, this quantity can be interpreted as the probability for a photon to be lost by the atom.

C. Slow light

We have evidenced that a one-dimensional atom is a highly dispersive medium. In particular, a quantum-dot cavity system evanescently coupled to a waveguide has a behavior similar to a medium showing dipole induced transparency. As a consequence, this optical system could be used to slow down photons. Let's consider the case of a cavity perfectly connected to a waveguide ($Q/Q_0 = 1$) containing a leaky quantum dot. The transmission coefficient in amplitude can be written $t = |t|e^{-i\phi_t(\omega)}$, where $\phi_t(\Delta\omega)$ varies near ω_0 on a scale Γ . We send in the optical system a wave packet $\psi_{in}(\omega)$ of width W centered around ω_0 . Denoting θ the temporal coordinate, we obtain the shape of the output pulse

$$\psi_{out}(\theta) \propto \int d\omega \psi_{in}(\omega) e^{i\omega\theta} e^{i\phi_t(\omega)}. \quad (35)$$

If the width of the wave packet fulfills $W \ll \Gamma$, we can develop ϕ_t around ω_0 . We finally obtain $\psi_{out}(\theta) = \psi_{in} \left[\theta - \left(\frac{\partial \phi_t}{\partial \omega} \right)_{\omega_0} \right]$. The wave packet will then be transmitted by the

optical system after a delay T_D which reads

$$T_D = \left(\frac{\partial \phi_t}{\partial \omega} \right)_{\omega_0} . \quad (36)$$

During the transmission the wave packet will also be damped by a factor $T = |t|^2$. Remembering that $\kappa \gg \Gamma$, we neglect the variations due to the cavity mode. We shall then take $t'_0(\Delta\omega) \sim 1$ and

$$t \sim \frac{f}{1+f} \frac{1}{1 + \frac{2i}{\Gamma} \Delta\omega \frac{Q}{Q_0} \frac{f}{1+f}} . \quad (37)$$

With this hypothesis, $\phi_t \sim \arctan\left(\frac{f}{1+f} \frac{2\Delta\omega}{\Gamma}\right)$. As a consequence,

$$T_D \sim \frac{2}{\Gamma} \arctan'(x)_0 \sim \frac{2}{\Gamma} \frac{f}{1+f} . \quad (38)$$

The wave packet is delayed by the lifetime of the dipole, which was expected. The damping factor has the following form

$$T = \left(\frac{f}{1+f} \right)^2 \quad (39)$$

This process could be repeated using a series of N optical devices. We note $N_{1/2}$ the number of devices such that the outcoming power is half the incoming one. $N_{1/2}$ checks

$$N_{1/2} = \frac{1}{2} \frac{\log 2}{\log(1 + 1/f)} \quad (40)$$

Supposing f sufficiently high, we have $\log(1 + 1/f) \sim 1/f$ and $N_{1/2}$ scales like f . We could finally obtain a delay \mathcal{T}_D

$$\mathcal{T}_D = N_{1/2} \frac{2}{\Gamma} \frac{f}{1+f} \propto \frac{\Gamma}{2} f \quad (41)$$

In particular, we could use a series of microdisks each evanescently coupled to the same wave guide. This generalizes the study of Heebner et al [17] who have shown that the group velocity of a signal passing through a series of empty microdisks scales like the inverse of the finesse of the resonators.

IV. NON-LINEAR CASE

We are now interested in the behavior of the optical system for arbitrary intensities of the incoming field. In a first step, we shall establish the optical Bloch equations for the two-level system interacting with a coherent field injected in the input mode, in the ideal case of the "one-dimensional atom" where the leaks can be neglected. We will adopt the semi-classical hypothesis where the quantum correlations between atomic operators and field operators can be neglected. We shall comment the range of validity of this approximation at the end of the section. In a second step we will focus on the influence of the leaks on the expected visibility of the experimental signals.

A. Giant optical non-linearity

We take the mean value of equations (13), and we suppose $\langle b'_{in} \rangle = 0$. Note that $\langle b_t \rangle$ and $\langle b_r \rangle$ could be measured using a homodyne detection. Writing $s = \langle S_- \rangle$, $s_z = \langle S_z \rangle$, and identifying b_t (respectively b_r) to $\langle b_t \rangle$ (respectively to $\langle b_r \rangle$) we obtain

$$\begin{aligned} \dot{s} &= -i\Delta\omega s - \frac{\Gamma}{2}t_0(\Delta\omega)s + i\sqrt{\frac{\Gamma}{2}}(-2s_z)b_{in}t_0(\Delta\omega) \\ \dot{s}_z &= -\Gamma\Re(t_0(\Delta\omega))\left(s_z + \frac{1}{2}\right) + \sqrt{\frac{\Gamma}{2}}(is^*b_{in}t_0(\Delta\omega) + cc) \\ b_t &= -\left(b_{in} + i\sqrt{\frac{\Gamma}{2}}s\right)t_0(\Delta\omega) \\ b_r &= b_{in} + b_t. \end{aligned} \tag{42}$$

Equations (42) are similar to the well known Bloch optical equations for a two-level system interacting with a classical field with a coupling constant Γ . Nevertheless, in this case the dipole relaxation rate is related to the coupling constant, whereas usually the two parameters are independant. This is due to the fact that the dipole is driven and relaxes via the same ports 1 and 2. We obtain after some little algebra detailed in appendix C the stationary solution for the population of the two-level system

$$\begin{aligned} s &= \sqrt{\frac{2}{\Gamma}} \frac{1}{1+x} \frac{ib_{in}}{1 + \frac{2i\Delta\omega}{\Gamma t_0(\Delta\omega)}} \\ s_z &= -\frac{1}{2} \frac{1}{1+x}, \end{aligned} \tag{43}$$

where we have introduced the saturation parameter x

$$x = \frac{|b_{in}|^2}{P_c(\Delta\omega)}. \quad (44)$$

$P_c(\Delta\omega)$ is the critical power necessary to reach $s_z = -1/4$, satisfying

$$\begin{aligned} P_c(\Delta\omega) &= \frac{\Gamma}{4}\phi(\omega) \\ \phi(\omega) &= \left(\frac{2\Delta\omega}{\Gamma}\right)^2 + \left(\frac{2\Delta\omega}{\Gamma}\frac{\Delta\omega + \delta}{\kappa} - 1\right)^2. \end{aligned} \quad (45)$$

P_c scales like a number of photons per second. At resonance it corresponds to one forth of photon per lifetime. Out of resonance it is increased by a factor $\phi(\omega)$ which can be seen as the inverse of an adimensional cross-section.

We define an adimensional susceptibility α for the two-level system

$$s = \sqrt{\frac{2}{\Gamma}}\alpha b_{in}, \quad (46)$$

where α reads

$$\alpha = \frac{1}{1+x} \frac{i}{1 + \frac{2i\Delta\omega}{\Gamma t_0(\Delta\omega)}}. \quad (47)$$

We have plotted in figure 5 the evolution of the real and imaginary part of the susceptibility as a function of $\omega - \omega_0 = -\Delta\omega$ for different values of the saturation parameter. As it can be seen on the figure, the non-linear effect is maximal at resonance, with

$$\alpha = \frac{i}{1+x}. \quad (48)$$

At resonance α is purely imaginary : the field is entirely absorbed by the dipole. Note that the behavior of the two-level system drastically changes from $|b_{in}|^2 \sim 0$ to $|b_{in}|^2 \sim 10P_c$. Any two-level system is then a giant optical non-linear medium. This well-known result was not exploited experimentally until now, because it is quite difficult to optically address a single two-level system. In particular, the fluorescence field emitted by a dipole is most of the time spread in all directions of space and lost for the observer, preventing him from operating on resonance. The "one-dimensional atom" geometry we aim to adopt allows to face these difficulties. Indeed the two-level system only exchanges energy with the cavity mode, while the cavity mode is mainly connected to input and output fields. As a consequence any

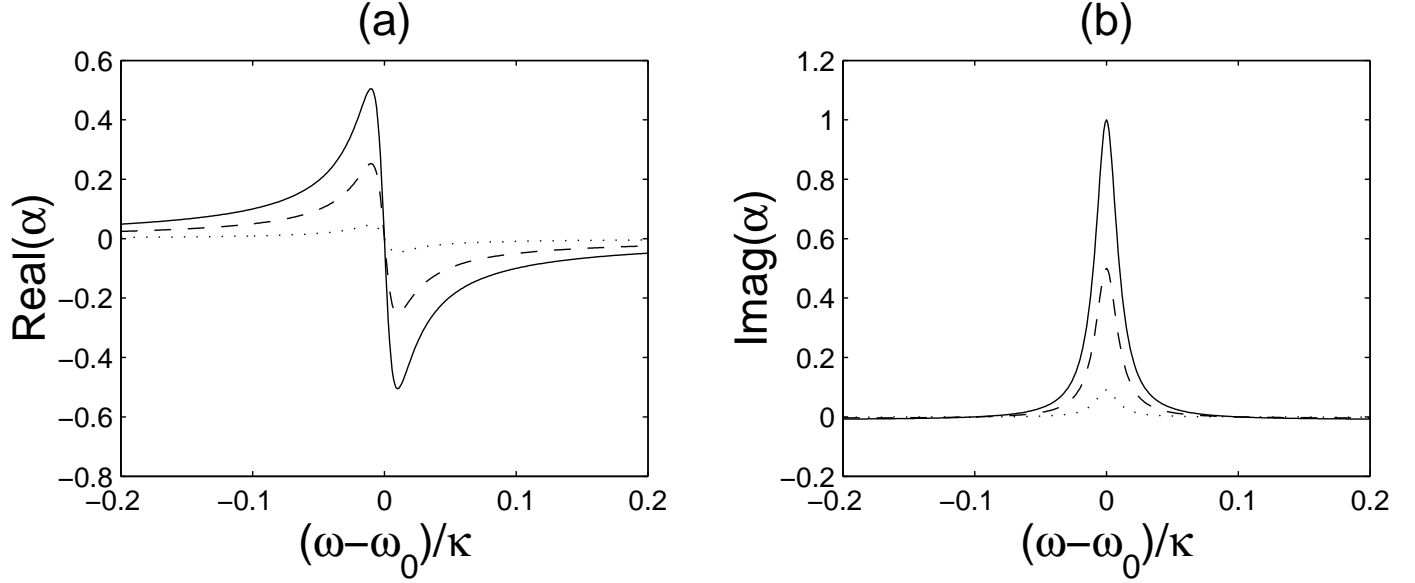


FIG. 5: Susceptibility α of the atomic dipole as a function of $(\omega - \omega_0)/\kappa = -\Delta\omega/\kappa$, for different values of the saturation parameter x . (a) Real part of α . (b) Imaginary part of α . Solid : $x=0$. Dashed : $x=1$. Dots : $x=10$.

photon in the input field will reach the two-level system, and the fluorescence field emitted by the two-level system will be observable in the output field. This is evidenced by the specific constitutive relation of our optical system obtained at resonance :

$$b_t = -b_{in}t_0(\Delta\omega)(1 + i\alpha) . \quad (49)$$

The input field interferes with the fluorescence field emitted by the dipole. Given that there are no leaks, the interference may be totally destructive at resonance as underlined in section III.

We have represented figure 6 the transmission coefficient $T = |t(\Delta\omega)|^2$ for different values of the incoming power. For low values the system is not saturated and the dipole blocks the light. For $|b_{in}|^2 = P_{in} > 10P_c$ the dipole is saturated and cannot prevent light from crossing the cavity. This non-linear behavior is obvious if we restrict ourselves to the resonant case.

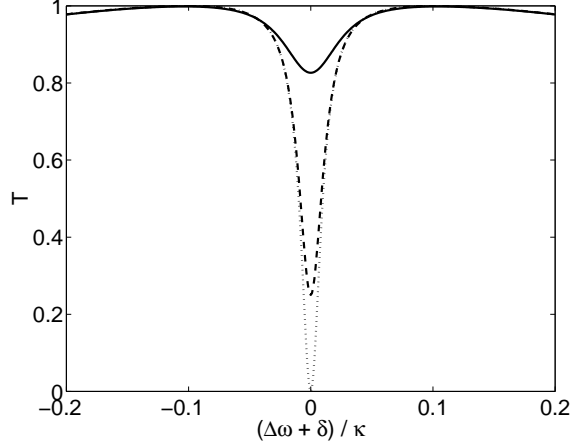


FIG. 6: *Transmission of the optical system as a function of the normalized detuning $(\Delta\omega + \delta)/\kappa$ between the quantum dot and the driving frequency for different values of saturation parameter at resonance $x = 4|b_{in}|^2/\Gamma$. We took $\delta = 0$ for convenience. Dots : $x = 0$. Dashed-dot : $x = 1$. Solid : $x = 10$.*

At resonance indeed the transmission and reflection coefficients in amplitude t and r write

$$\begin{aligned} t &= \frac{-x}{1+x} \\ r &= \frac{1}{1+x}, \end{aligned} \quad (50)$$

which implies for the transmitted and reflected power P_t and P_r

$$\begin{aligned} P_t &= \frac{x^2}{(1+x)^2} P_{in} \\ P_r &= \frac{1}{(1+x)^2} P_{in}. \end{aligned} \quad (51)$$

R , T , P_r and P_t are plotted in figure 7. As expected a non-linear jump in the transmission coefficient happens at a typical power for the incoming field $P_{in} \sim P_c/2$. At this point the slope of the function $T(x)$ reaches $(2/3)^3$ which is a maximum. Note that this giant optical non-linearity has been pointed out in the case of a two-level system in an asymmetric cavity [10], the non-linear jump being observable in the phase of the reflected field.

It should be noticed at this point that $P_r + P_t \neq P_{in}$ even for an ideal non-leaky system as considered in this subsection. To understand this, let us remind that $P_t + P_r$ is the power of the coherently diffused field, which is predominant for a low power of the driving field. On the contrary, when the dipole is saturated, the fluorescence field is emitted with a random

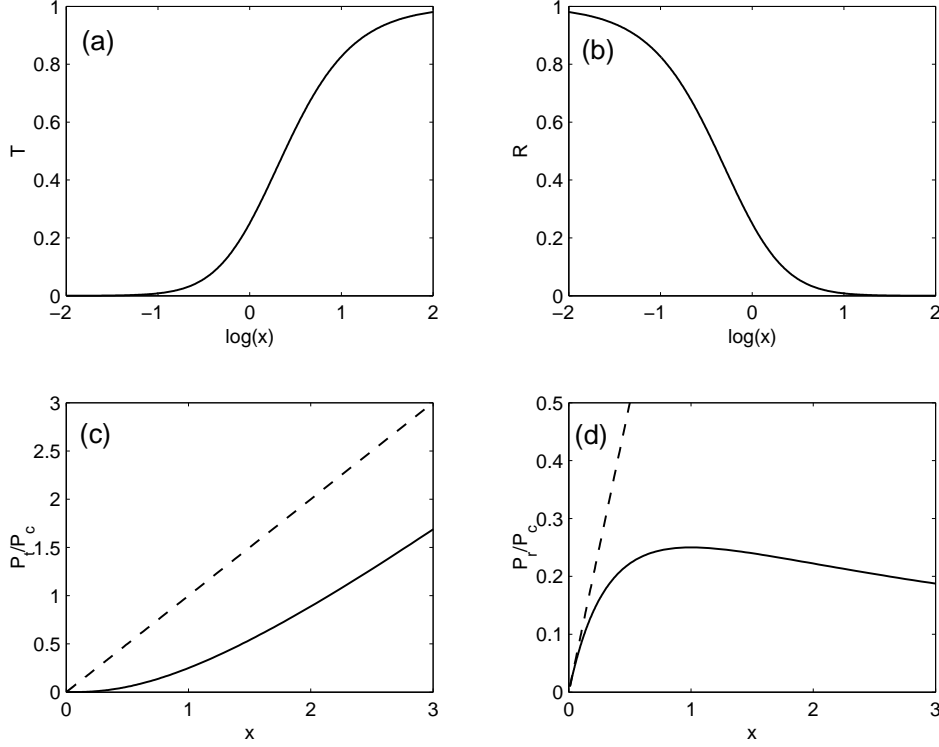


FIG. 7: (a) Transmission, (b) reflection coefficient as a function of the logarithm of the saturation parameter on resonance $\log(x) = \log(4|b_{in}|^2/\Gamma)$. (c) Solid : normalized transmitted power P_t/P_c and (d) normalized reflected power P_r/P_c as a function of the saturation parameter x . Dashed : normalized incoming power P_{in}/P_c . The transmitted field corresponds to the driving field lowered by one photon per lifetime, which has been absorbed by the atom. This reflected field increases with the driving field until $x = 1$, and then decreases because of the saturation of the two-level system.

phase and cannot interfere with the driving field anymore [10, 20]. This incoherent diffusion process is responsible for a noise whose power P_{noise} allows to preserve energy conservation

$$P_{noise} = P_{in} - P_r - P_t \sim \frac{2x}{(1+x)^2} P_{in} . \quad (52)$$

Let's mention here that P_{noise} would be detected with direct photon counting and would be split between the two output ports. We have plotted in figure 8 the relative contribution of the noise power P_{noise} and of the coherently diffused fields $P_r + P_t$ over the incoming power P_{in} , as a function of the logarithm of the saturation parameter. The noise contribution is maximal for $x = 1$. This also gives us a glimpse of the range of validity for the semi-classical

assumption, which correctly describes the problem only out of the non-linear jump.

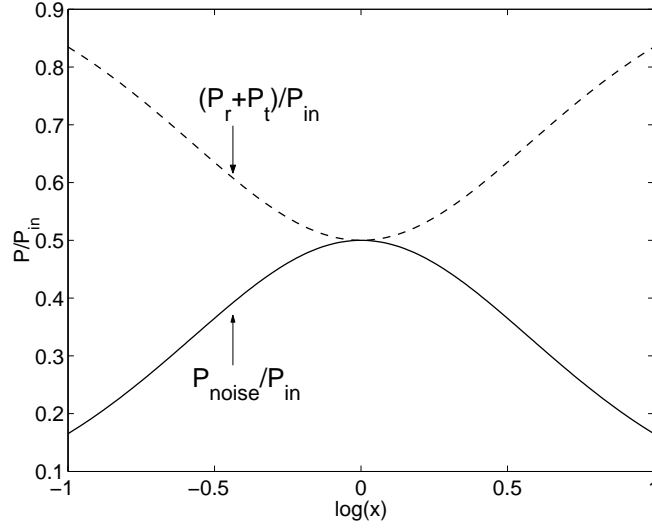


FIG. 8: $\log(P/P_{in})$ as a function of $\log(x)$. Solid : $\log(P_{noise}/P_{in})$. Dashed : $\log(P_r + P_t/P_{in})$.

B. Influence of the leaks

We consider now the case of a leaky optical system described by equations (23). We shall restrict ourselves to the resonant case and to the semi-classical hypothesis. For sake of simplicity we shall also take $\gamma^* = 0$, which is a realistic hypothesis as it will be shown in the next section. As before we can adiabatically eliminate the cavity from the equations. With $\langle b'_{in} \rangle = 0$, and using the same definitions for Γ , f , Q and Q_0 , we can establish the optical Bloch equations for the leaky system

$$\begin{aligned}
 \dot{s} &= -\frac{\Gamma}{2} \frac{Q}{Q_0} \left(1 + \frac{1}{f}\right) s + \sqrt{\frac{\Gamma}{2}} \frac{Q}{Q_0} (-2s_z) i b_{in} \\
 \dot{s}_z &= -\Gamma \frac{Q}{Q_0} \left(1 + \frac{1}{f}\right) \left(s_z + \frac{1}{2}\right) + \sqrt{\frac{\Gamma}{2}} \frac{Q}{Q_0} (i b_{in} s^* + cc) \\
 \dot{b}_t &= -b_{in} \frac{Q}{Q_0} - i \sqrt{\frac{\Gamma}{2}} \frac{Q}{Q_0} s \\
 \dot{b}_r &= b_{in} \left(1 - \frac{Q}{Q_0}\right) - i \sqrt{\frac{\Gamma}{2}} \frac{Q}{Q_0} s.
 \end{aligned} \tag{53}$$

At it is shown in appendix C, the stationary solutions can be written

$$\begin{aligned} s_z &= -\frac{1}{2} \frac{1}{1+x'} \\ s &= \sqrt{\frac{\Gamma}{2}} \frac{ib_{in}}{1+x'} \frac{1}{1+\frac{1}{f}}. \end{aligned} \quad (54)$$

with modified values for the saturation parameter x' and the critical power P'_c

$$\begin{aligned} x' &= |b_{in}|^2 / P'_c \\ P'_c &= \frac{\Gamma}{4\beta^2}. \end{aligned} \quad (55)$$

We have introduced the parameter $\beta = \frac{f}{1+f}$. The quantity β^2 can be seen as the probability for a resonant photon sent in the input mode to be absorbed by the optical system. The power necessary to saturate the two-level system, that is to reach $s_z = -1/4$, is higher than in the ideal case which is a natural consequence of the leaks. The transmission coefficient in energy can be written

$$T = \left(\frac{Q}{Q_0} \right)^2 \left[\frac{\beta}{1+\beta^2 x} - 1 \right]^2. \quad (56)$$

We have plotted in figure 9 the transmission coefficient T as a function of the saturation parameter in the non-leaky case $x = 4P_{in}/\Gamma$. The limit of the signal for $x \rightarrow 0$ is T_{min} because the two-level system is not saturated. If $x \rightarrow \infty$ the signal tends to T_{max} : if the two-level system is saturated, the optical system behaves like an empty cavity. On the left, we fixed $\beta = 1$ which may be realized with high values of the ratio f , and we considered different leaky cavities. In this case, the transmission coefficient simply corresponds to the ideal transmission coefficient multiplied by $(Q/Q_0)^2$. On the right, we have considered a non-leaky cavity ($Q/Q_0 = 1$) and different values of the ratio f . The jump happens for higher values of the saturation parameter, which was expected. It can be shown that the slope of the transmission coefficient is bounded by $(2/3)^3$ in any case, which has consequences for potential applications of this non-linearity as it will be shown in section VI.

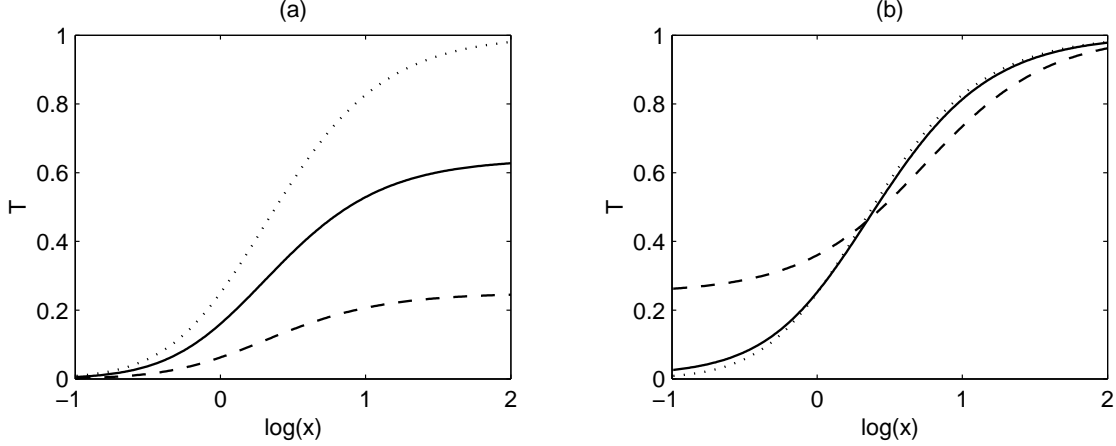


FIG. 9: Transmission of the optical system on resonance as a function of the logarithm of the saturation parameter $\log(x) = \log(4P_{in}/\Gamma)$. (a): We fixed $\beta = 1$ which corresponds to high values of f . Dots : $Q = Q_0 = 1000$ (ideal case). Solid : $Q = 800$. Dashed : $Q = 500$. The obtained signals are the ideal signal multiplied by $(Q/Q_0)^2$. (b): We took $Q = Q_0 = 1000$. Dots : ideal case. Dashed : $f = 1$. Solid : $f = 10$. The non-linear jump happens for higher values of the saturation parameter.

C. Quantifying the giant non-linearity

As underlined before, the non-linearity is giant because of two main effects, which are characteristics of the "one-dimensional atom" geometry: first any photon that is sent in the input field reaches the single two-level system; second, the fluorescence field is entirely directed in the output ports, so that there are no leaks and we can operate at resonance. To quantify the non-linearity it is convenient to observe that the transmission and reflexion jumps could be obtained using an optical medium inducing a non-linear phase jump of π without absorption, the jump happening for a typical intensity $I_\pi \sim 10P_c/\sigma$ where σ is the surface on which light is focused and the factor of 10 is evaluated from figure 13. Let's compute the typical intensity in our case. The critical power P_c is one forth photon per lifetime, that is, with a wavelength $\lambda \sim 1\mu m$ and a lifetime $\tau \sim 100$ ps, $P_c \sim 1$ nW. We shall take $\sigma \sim 10^{-8}$ cm² which corresponds to the surface of a micropillar. We obtain $I_\pi \sim 1W/cm^2$ Let's consider a non-linear Kerr medium with a refractive index given by $n = n_0 + n_2I$ where I is the intensity of the light beam crossing the medium. The non-linear

phase-shift acquired by the beam is

$$\phi_{nl} = \frac{2\pi}{\lambda} L n_2 I. \quad (57)$$

Given that the non-linear index of bulk semiconductor (like GaAs) at half gap excitation is typically $n_2 = 10^{-13} \text{ cm}^2/\text{W}$ [21], the length of medium should be 5.10^3 km to reach a π phase shift with the same intensity ! Resonant experiment using an atomic vapor in low finesse cavity have reached values of $n_2 \sim 10^{-7} \text{ cm}^2/\text{W}$ while preserving a quantum noise limited operation [22] : a π phase shift could be obtained after 5 m of vapor. More recently there has been work on slow light using electromagnetically induced transparency exhibiting giant resonant non linear refractive index $n_2 = 0.18 \text{ cm}^2/\text{W}$ [23], leading to a length of a few mm to reach the same effect.

V. FEASIBILITY STUDY

In this section, we aim at optimizing the parameters of a micropillar in order to have a maximally contrasted signal. Micropillars are very good candidates for this application because the light they emit is directional. As a consequence they have already been used with success as single photon sources [5, 6] and indistinguishable photon sources [7].

A. Optimization

We can experimentally control two parameters: the intrinsic quality factor Q_0 and the diameter d of the micropillar. Q_0 corresponds to the quality factor of the planar cavity and is tunable by changing the reflectivity of each Bragg mirror. The diameter d is adjusted during the lithography and etching step. The total quality factor Q of the micropillar reads

$$\frac{1}{Q} = \frac{1}{Q_0} + \frac{1}{Q_{leak}}, \quad (58)$$

where the leaks are mainly due to the etching step and can be written [24]

$$\frac{1}{Q_{leak}} = \frac{2|E(d)|^2 \varepsilon}{d}. \quad (59)$$

$|E(d)|$ is the electrical field of the fundamental mode at the sidewalls of the micropillar, whose profile is given by the Bessel function of the first kind J_0 [24]. The parameter ε is a

parameter quantifying the etching quality. The leaks increase as the diameter of the etched micropillar decreases. In the following we will take $\varepsilon \sim 0.007$ which corresponds to realistic experimental parameters [25].

The experimental signal to maximize is defined as $\mathcal{C} = T_{max} - T_{min}$, where T_{max} and T_{min} are given by equations (30) and (31), $T_{max} = (Q/Q_0)^2$ and $T_{min} = (Q/Q_0)^2(1/(1+f))^2$. We have chosen to optimize an amplitude rather than a visibility $\mathcal{V} = (T_{max} - T_{min})/(T_{max} + T_{min})$ because we should be then less sensitive to the optical background. Given that we use quantum dots as two-level system we should take into account the excitonic dephasing γ^* . Nevertheless, some experimental conditions have been shown to drastically reduce this parameter [26]. In particular, excitonic dephasing times limited by radiative recombination have already been observed for a resonant excitation of the fundamental optical transition of InAs quantum dots at low temperature. Therefore, excitonic dephasing might affect only marginally as well the system under study [26, 27].

A first strategy to optimize the contrast \mathcal{C} is to reach small T_{min} , that is high Purcell factor F_p , where F_p can be written [28]

$$F_p = \frac{3Q}{4\pi^2 V} \left(\frac{\lambda}{n} \right)^3. \quad (60)$$

The quantity λ is the dipole wavelength in the vacuum, n the refractive index of the medium and V the effective volume of the mode,

$$V \sim \left(\frac{\lambda}{n} \right) \frac{\pi d^2}{8}. \quad (61)$$

Figure 10 represents the evolution of Q and F_p as a function of the micropillar diameter for three different values of the intrinsic quality factor Q_0 : 1000, 5300 and 10000. If the diameter is too small, the leaks degrade Q and as a consequence F_p . If the diameter is too large, F_p decreases because of the large modal volume. The optimal diameter varies between 1 and 2 μm . As it can be seen on the figure, a higher initial Q_0 allows to reach higher values of F_p , and corresponds to higher optimal diameters.

At the same time we need high T_{max} , which corresponds to small cavity leaks and to large diameters. We have represented figure 11 the evolution of $T_{max} - T_{min}$ as a function of the micropillar diameter for different Q_0 . As expected, the optimal diameters are higher than the ones obtained by optimization of Purcell factor, and vary now between 2 and 6 μm . For

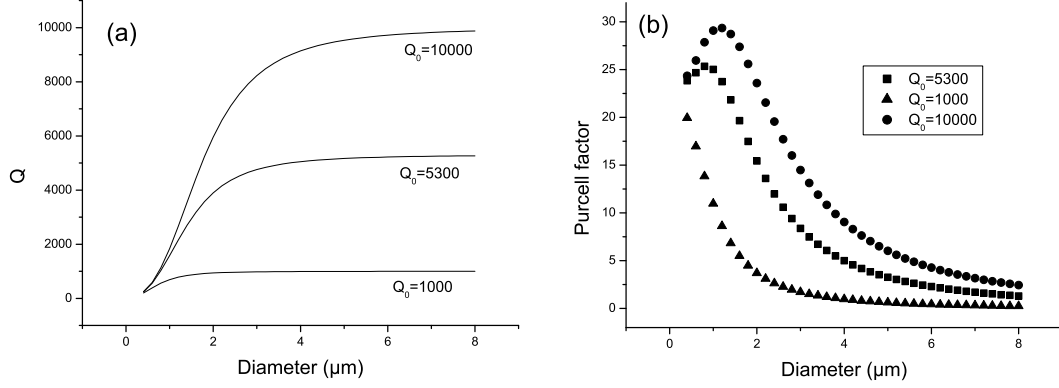


FIG. 10: *Quality factor of a micropillar cavity (a) and Purcell factor of a single quantum dot in the cavity mode (b) as a function of the diameter of the micropillar, for different Q_0 factors of the intrinsic cavity. Squares : $Q_0 = 5300$. Dots : $Q_0 = 10000$. Triangles : $Q_0 = 1000$. We took $\frac{1}{Q} = \frac{1}{Q_0} + \frac{1}{Q_{leak}}$ with $\frac{1}{Q_{leak}} = \frac{2|E(d)|^2 \varepsilon}{d}$. The quantity $|E(d)|$ is the electrical field at the sidewalls of the micropillar. The parameter ε quantifies the leaks due to the etching. We took here $\varepsilon = 0.007$.*

each Q_0 , the amplitude of the optimized signal is higher than 0.8 which is quite convenient. We shall prefer the set of parameters corresponding to the smallest diameter, so that it is easier to isolate a single quantum dot, that is $Q_0 = 1000$, $d = 2.4 \mu\text{m}$, $Q = 960$ and $F_p = 2.6$. The expected amplitude of the experimental signal would then be 0.85.

We shall mention here another strategy to enhance f , that consists in reducing the leaks γ_{at} . Recent experiments involving the metallization of the micropillars have shown a reduction of γ_{at} by a factor 10 [29]. The expected T_{min} obtained with such a metallized cavity should be under 10^{-3} , and the signal amplitude near 0.9.

To have a glimpse of the expected signal we have plotted in figure 12 the transmission of the system as a function of the detuning between the atom and the field for $Q_0 = 1000$, $Q = 500$ and $F_p = 3$ (dot curve) which corresponds to realistic parameters for single photon sources before optimization [6]. The contrast of the signal is 0.21. On the same figure we

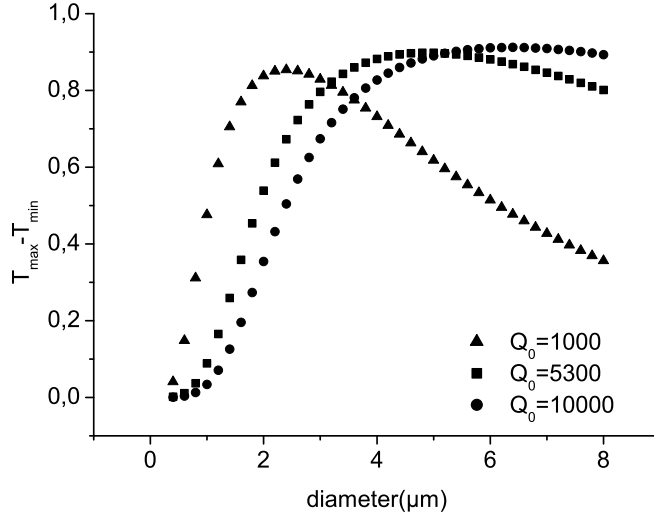


FIG. 11: *Amplitude of the signal $T_{max}-T_{min}$. Squares : $Q_0 = 5300$. Dots : $Q_0 = 10000$. Triangles: $Q_0 = 1000$. Amplitudes as high as 0.9 can be obtained using state of the art microcavities.*

have also plotted the expected signal after optimization of the micropillar, with and without metallization of its sidewalls.

We have finally plotted in figure 13 the transmission coefficient on resonance as a function of the logarithm of the saturation parameter $\log(x)$ for these three different sets of parameters. We shall be able to observe the non-linear transmission jump with state-of-the-art micropillar cavities.

B. Discussion on some merit figures of the optical system

We have seen that the "one-dimensional atom" case requires $f \rightarrow \infty$ and $Q/Q_0 \rightarrow 1$. Such an optical system would also provide a high efficiency single-photon source. The expression of the raw quantum efficiency η [30] of these devices reads indeed

$$\eta = \frac{f}{1+f} \frac{Q}{Q_0}. \quad (62)$$

The prefactor $\beta = f/(1+f)$ has been introduced in section IV, it represents the fraction of photons spontaneously emitted by the excited atom into the cavity mode, whereas Q/Q_0 is the fraction of photons initially in the cavity mode finally funneled into the mode(s) of

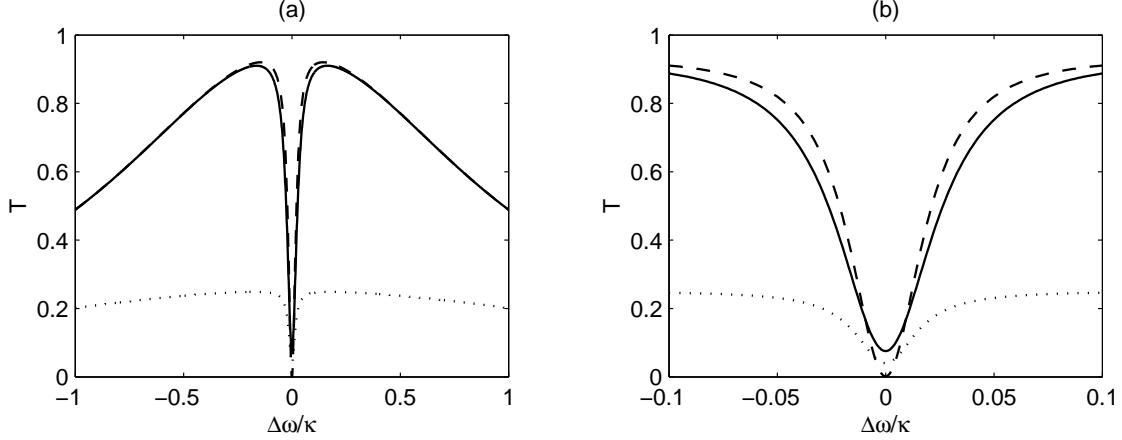


FIG. 12: (a) *Transmission of the optical system as a function of the normalized detuning $\Delta\omega/\kappa$ between the quantum dot and the driving frequency. We took $\delta = 0$ for convenience. Dots : we took $Q_0 = 1000$, $Q = 500$, $F_p = 3$. Solid : $Q_0 = 1000$, $Q = 960$, $F_p = 2.6$, $d = 2.4 \mu\text{m}$ which are the parameters resulting from the optimization of the micropillar. Dashed : Same parameters after metallization of the sidewalls of the micropillar. (b) Zoom on the dips.*

interest. Note that usually for single-photon sources, the photons are collected in only one output mode. In the present case, transmitted *and* reflected photons must be collected to measure the quantum efficiency of the corresponding source. One may ask if optimizing the system as a single photon source is equivalent to optimizing it as a medium providing dipole induced transparency (one should then maximize the visibility \mathcal{C} of the signal) or as a giant non-linear medium (this would require a low critical power, that is a high absorption probability β^2 as it has been introduced in section IV). We have plotted in figure 14 the parameters \mathcal{C} , β^2 and η as functions of the diameter of the micropillar for an initial quality factor $Q_0 = 1000$. As it can be seen on the figure, optimal diameters are different. The optimization of \mathcal{C} leads to the highest diameter. As it is explained in paragraph A, this is because non-leaky cavities (and as a consequence high diameters) are needed to reach high T_{max} . It is striking to observe that β^2 and η have different evolutions. Indeed one could have thought that a good single-photon source, that is an optical system that emits photons with high efficiency in a particular mode, is also able, when it is driven by a resonant field, to absorb and reemit photons with high efficiency. Yet β^2 is optimized for smaller diameters than η : the absorption probability is more sensitive to the atomic leaks than the single

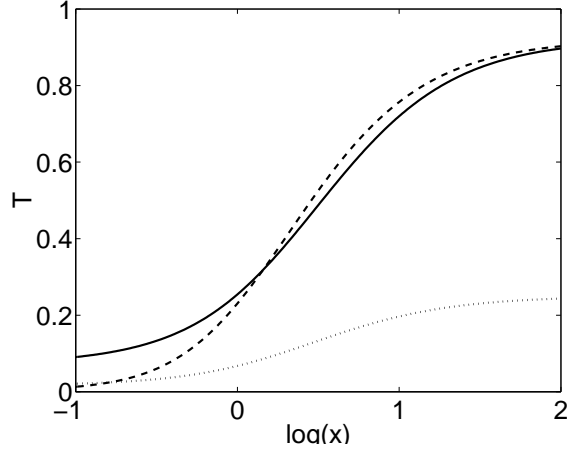


FIG. 13: *Transmission of the optical system as a function of the logarithm of the saturation parameter $\log(x) = \log(4P_{in}/\Gamma)$. Dots : we took $Q_0 = 1000$, $Q = 500$, $F_p = 3$. Solid : $Q_0 = 1000$, $Q = 960$, $F_p = 2.6$, $d = 2.4 \mu\text{m}$ which are the parameters resulting from the optimization of the micropillar. Dashed : Same parameters after metallization of the sidewalls of the micropillar.*

photon source efficiency. Even if the cavity is leaky, an atom perfectly connected to the cavity mode can absorb one photon in the input field with a maximal probability. The major difference between these two behaviors is that they are observed in two quite different regimes. The quantity η is the probability of detecting a photon in the mode of interest *conditionned to the excitation of the atom*, and can be computed by supposing that in a first step, the atom has emitted a photon in the cavity mode, and that in a second step, this photon has been funneled into the mode of interest. On the contrary, β^2 is estimated in a permanent regime where the driving field can interfere with the fluorescence field as pointed out in section III. Because of this interference phenomenon, it is impossible to describe the evolution of the photon by successive interactions with the cavity mode and with the atom : the atom-cavity coupled system must be considered as a whole. A signature of this interference effect has been observed in section III, where total reflection was induced by an atom perfectly connected to a leaky cavity.

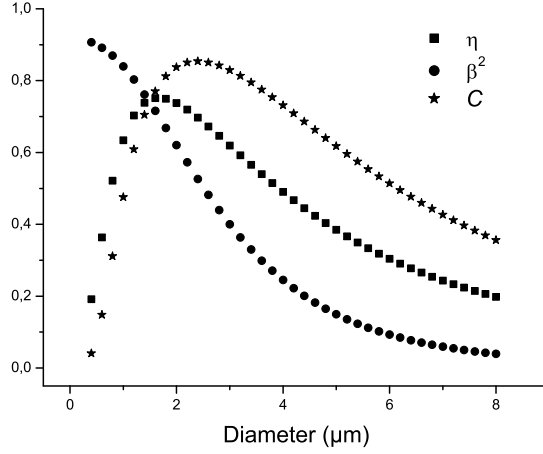


FIG. 14: *Characteristics of the quantum dot-cavity system as a function of the diameter of the micropillar. We took $Q_0 = 1000$. Squares: raw quantum efficiency η . Stars: expected amplitude C of the experimental signal. Dots : probability of photon absorption β^2 . The contrast C is more sensitive than η to the leaks of the cavity, leading to higher optimal diameters.*

VI. PERSPECTIVES ON PHOTONIC COMPUTATION

A. Classical computation

1. About optical switches

Looking at the transmission coefficient, we could be tempted to use the giant non-linearity to realize an all-optical switch. Bistability regime is expected to be quite useful with this aim [31, 32]. As it is represented on figure 15, we could for example re-inject part of the transmitted intensity in the input port to realize a bistable device. Unfortunately the slope of the signal is too low. Calling P_0 the signal coming in the loop, P_e the signal entering the device, P_t the power transmitted by the device and A the fraction of P_t used to create the bistability, we have

$$P_e = P_0 + AP_t(P_e) . \quad (63)$$

Bistability happens for values of the parameter B for which equation $P_0(P_e) = B$ has more than one solution. At low intensity $P_t \sim 0$ and $P_e \sim P_0$. At high intensity $P_t \sim P_0$

and $P_e \sim (1 - A)P_0$. The system will exhibit bistability if P_0 decreases as P_e increases. This can only be done if $\partial P_t / \partial P_e > 1$. Nevertheless, as it has been shown in section IV, the slope of the signal $P_t(P_e)$ is bounded by $(2/3)^3 < 1$, preventing the system from reaching the bistability regime.

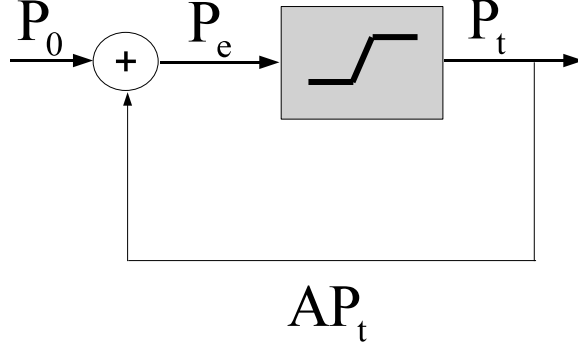


FIG. 15: *Scheme of a possible use of the optical system to generate bistability. Part of the transmitted power is reinjected at the entrance of the device. The weak slope of the function $T(P_{in})$ doesn't allow to reach the bistability regime.*

2. Reshaping step

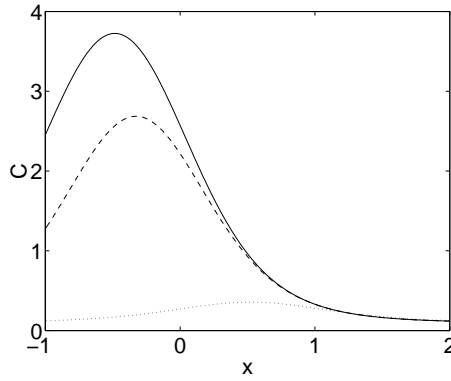


FIG. 16: *Contrast enhancement ratio as a function of the saturation parameter $x = 4P_{in}/\Gamma$ for different values of f . $Q_0 = 1000$, $Q = 960$. Dot : $f = 2.6$. Dashed : $f = 50$. Solid : $f = 100$.*

A possible application is to use the non-linearity to enhance the contrast ratio between two pulses of different intensities. This can be used to regenerate optical signals travelling in

an optical fiber. The major advantage of this system compared to other devices is the very low switching energy, defined as the energy necessary to saturate the system and make it switch from a linear to a non-linear behavior. As already seen, the typical switching energy is $0.25 \, h\nu$ where $h\nu$ the energy of a resonant photon. We have $h\nu \sim 1 \, \text{eV} \sim 4.10^{-20} \, \text{J}$ which is 8 orders of magnitude lower than for traditional saturable absorbers [33]. The figure of merit for this kind of devices is the contrast enhancement ratio, defined as

$$\mathcal{C} = \left(\frac{P_L}{P_H} \right)_{in} \left(\frac{P_H}{P_L} \right)_t \quad (64)$$

where P_H (resp P_L) is the high-power pulse (resp the low one). The subscript in (resp t) describes the incoming field (resp transmitted). We introduce the extinction ration of the pulse

$$d = \left(\frac{P_H}{P_L} \right)_{in} . \quad (65)$$

For a perfect non-linear device, \mathcal{C} writes

$$\mathcal{C} = \frac{1}{d} \frac{T(x)}{T(x/d)} , \quad (66)$$

where T is the transmittance at resonance of the device. We have

$$\mathcal{C} = d \left(\frac{1+x}{1+x/d} \right)^2 , \quad (67)$$

where \mathcal{C} is maximum for $x \rightarrow 0$ and tends to d . With an ideal device we could theoretically reach any value of \mathcal{C} . Taking into account the leaks, and denoting T_{leak} the transmission of the device on resonance, and \mathcal{C}_{leak} the new contrast enhancement factor, we obtain

$$\mathcal{C}_{leak} = \frac{1}{d} \frac{T_{leak}(x)}{T_{leak}(x/d)} . \quad (68)$$

We have represented figure 16 the contrast enhancement factor for different values of the factor f which has been defined in equation (28). Let's recall that f is related to the Purcell factor by the simple expression $f = (\gamma_{free}/\gamma_{at})F_p$ if there is no excitonic dephasing. The intrinsic quality factor (resp. the quality factor) of the cavity has been taken equal to 1000 (resp. 950). The extinction ratio is doubled for $f \sim 30$, which corresponds to a typical Purcell factor of 3, and $\gamma_{at}/\gamma_{free} \sim 0.1$ which could be obtained by metallizing the sidewalls

of a micropillar cavity as it has been underlined in section V. The ratio increases with f , a 6 *dB* enhancement is reached for $f \sim 100$ which is within reach of the micropillar or photonic crystal technology.

B. Quantum computation

There has been a considerable number of proposals, e.g. [34, 35, 36], and experiments, e.g. [19, 37], concerning the use of single emitters in high-finesse cavities for quantum information processing. Most of these papers are based on achieving the strong coupling regime. A recent proposal relying on the Purcell regime [11] requires the coherent control of additional levels in the emitter. It is natural to ask whether the most basic non-linearity considered in the present paper could be used directly for quantum information applications, for example for implementing a controlled phase gate between two photons, as suggested in Refs. [8, 10]. Unfortunately recent results suggest that this may not be possible. A numerical study [16] found fidelities of quantum gates employing the present non-linearity of order 80 %, which is quite far from what would be desirable for quantum computing or even quantum communication. Higher fidelities are elusive because the interaction with the single two-level system introduces temporal correlations between the two input photons. An analogous difficulty is discussed in detail in a recent theoretical paper on the use of Kerr non-linearities for quantum computing [38]. From a quantum information perspective the relatively simple situation considered in the present paper may thus best be seen as an important step towards the realization of more complex configurations.

VII. CONCLUSION

We have shown that a single two-level system in Purcell regime is a medium with appealing non-linear optical properties. In the linear case the two-level system prevents light from entering the cavity : this is dipole induced reflectance. This property vanishes as soon as the two-level system is saturated, which happens for very low power, of the order of one photon per lifetime (typically 1nW). As a consequence, such a medium shows a sensitivity at the single-photon level. We have established the optical Bloch equations describing this behavior in the semi-classical context, and shown that signatures of the non-linearity

should be observable using quantum dots and state-of-the-art semiconducting micropillars as two-level systems and cavities respectively. We have explored possible applications of the non-linearity in the context of photonic information processing.

VIII. ACKNOWLEDGMENTS

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APPENDIX A: DERIVATION OF EQUATION (14)

We show in this section that (b_r, b_t) and (b_{in}, b'_{in}) are related by a unitary transformation. For sake of completeness we keep the general form for θ_1 and θ_2 . Equations (13) can be written in the stationary linear case

$$S_- = \sqrt{\frac{2}{\Gamma}} \frac{e^{i\theta_1} b_{in} + e^{i\theta_2} b'_{in}}{1 + \frac{2i\Delta\omega}{\Gamma t_0(\Delta\omega)}}. \quad (\text{A1})$$

As a consequence, b_t reads

$$b_t = t_0(\Delta\omega) e^{i(\theta_1 - \theta_2)} \left(-1 + \frac{1}{1 + \frac{2i\Delta\omega}{\Gamma t_0(\Delta\omega)}} \right) b_{in} + \left(1 - t_0(\Delta\omega) + \frac{t_0(\Delta\omega)}{1 + \frac{2i\Delta\omega}{\Gamma t_0(\Delta\omega)}} \right) b'_{in}. \quad (\text{A2})$$

It can be rewritten in the following way

$$b_t = -\frac{e^{-i(\theta_2 - \theta_1)}}{1 + i\zeta} b_{in} + \frac{i\zeta}{1 + i\zeta} b'_{in}, \quad (\text{A3})$$

with

$$\zeta = \frac{\Delta\omega + \delta}{\kappa} - \frac{\Gamma}{2\Delta\omega}. \quad (\text{A4})$$

We easily compute b_r by switching b_{in} and b'_{in} , and θ_1 and θ_2 . We finally obtain

$$\begin{pmatrix} b_r \\ b_t \end{pmatrix} = \begin{pmatrix} \frac{i\zeta}{1+i\zeta} & \frac{-e^{i(\theta_2-\theta_1)}}{1+i\zeta} \\ \frac{-e^{-i(\theta_2-\theta_1)}}{1+i\zeta} & \frac{i\zeta}{1+i\zeta} \end{pmatrix} \begin{pmatrix} b_{in} \\ b'_{in} \end{pmatrix}. \quad (\text{A5})$$

The scattering matrix can be written in the following form

$$S = \frac{e^{i\phi}}{\sqrt{1+\zeta^2}} \begin{pmatrix} \zeta & ie^{i(\theta_2-\theta_1)} \\ ie^{-i(\theta_2-\theta_1)} & \zeta \end{pmatrix}, \quad (\text{A6})$$

with

$$\phi = \arctan\left(\frac{1}{\zeta}\right). \quad (\text{A7})$$

The S matrix is a unitary transformation up to a global phase. As a consequence energy is conserved by this transformation. Keeping in mind this property we shall rather use the form (A5) whose coefficients have a more direct physical interpretation.

APPENDIX B: DISCUSSION ON PHASE REFERENCE

In this appendix we show that θ_1 depends on the phase reference of the atomic dipole and of the incoming fields b_{in} . We write $s = \langle S \rangle$, $s_z = \langle S_z \rangle$, $b_{in} = \langle b_{in} \rangle$ and we suppose $\langle b'_{in} \rangle = 0$. The mean value of equations (13) reads

$$\dot{s} = -i\Delta\omega s + \frac{\Gamma}{2}st_0 - \sqrt{\frac{\Gamma}{2}}(-2s_z)e^{i\theta_1}b_{in}t_0. \quad (\text{B1})$$

We suppose $\delta = 0$ and $\Delta\omega \ll \kappa$ which is allowed by the fact that the cavity is much larger than the atom. We have $t_0 \sim -1$ and we obtain the following evolution equation for s

$$\dot{s} = -i\Delta\omega s - \frac{\Gamma}{2}s + \sqrt{\frac{\Gamma}{2}}(-2s_z)e^{i\theta_1}b_{in}. \quad (\text{B2})$$

We shall compare this equation with the one obtained from an ab-initio semi classical reasoning. The hamiltonian of a two-level system interacting with a classical electrical field $E(t) = E_0 \cos(\omega t - \phi)$ writes

$$H = \hbar\omega_0 S_z - E_0 D \cos(\omega t - \phi), \quad (\text{B3})$$

where D is the dipole operator. It can be written in the two-level system basis

$$D = d |e\rangle \langle g| + d^* |g\rangle \langle e| , \quad (\text{B4})$$

with $d = \langle e| D |g\rangle$. Traditionnaly d is taken real, which implies a choice of phase reference for the dipole. The Heisenberg equation for the dipole operator \hat{S}_- reads

$$\dot{\hat{S}}_- = \frac{1}{i\hbar} [\hat{S}_-, H] = -i\omega_0 \hat{S}_- + iE_0 d (-2S_z) \cos(\omega t - \phi) . \quad (\text{B5})$$

We write (B5) in the frame rotating at the drive frequency, and take the mean value of the resulting equation. Writing $s = \langle \hat{S} e^{i\omega t} \rangle$ we obtain

$$\dot{s} = -i(\omega_0 - \omega)s + i\frac{E_0 d}{2} (-2s_z) (e^{i\phi} + e^{2i\omega t - \phi}) . \quad (\text{B6})$$

We make the rotating wave approximation (RWA) consisting in neglecting the term varying at 2ω . Note that RWA is supposed in the Jaynes-Cummings hamiltonian used in this article. Comparing (B2) to (B6), we finally find

$$\phi + \frac{\pi}{2} = \theta_1 + \text{Arg}(b_1) . \quad (\text{B7})$$

We choose the time origin so that $\phi = 0$. If we take b_1 real, we obtain $\theta_1 = \pi/2$. The same reasoning leads to b_2 real and $\theta_2 = \pi/2$. We shall keep these values in the following.

APPENDIX C: DERIVATION OF THE CRITICAL INTENSITY INCLUDING LEAKS

In this section we derive the expression for the critical intensity in the non-resonant case in presence of leaks. We suppose $\gamma^* = 0$ for convenience. The stationary cavity population writes

$$a = t'_0 \frac{Q}{Q_0} \frac{-\Omega S_- + \sqrt{\kappa}(ib_{in} + b'_{in}) + H}{\kappa} , \quad (\text{C1})$$

where t'_0 has the following expression

$$t'_0 = \frac{1}{1 + i\frac{Q}{Q_0} \frac{\Delta\omega + \delta}{\kappa}} . \quad (\text{C2})$$

The semi-classical equations describing the evolution of s_z and s write

$$\begin{aligned}\dot{s} &= -i\Delta\omega s - \frac{\Gamma}{2} \frac{Q}{Q_0} \left[t'_0 + \frac{1}{f} \right] s - i \frac{Q}{Q_0} \sqrt{\frac{\Gamma}{2}} (2s_z) b_{in} t'_0 \\ \dot{s}_z &= -\Gamma \frac{Q}{Q_0} \left[\Re(t'_0) + \frac{1}{f} \right] \left(s_z + \frac{1}{2} \right) + \sqrt{\frac{\Gamma}{2}} \frac{Q}{Q_0} \left[i s^* b_{in} t'_0 + cc \right].\end{aligned}\tag{C3}$$

where f , Q and Q_0 have been defined in section III. By sake of completeness we also give the expressions for b_t and b_r after adiabatic elimination of the cavity mode

$$\begin{aligned}b_t &= -\frac{Q}{Q_0} t'_0 b_{in} - i \frac{Q}{Q_0} \sqrt{\frac{\Gamma}{2}} t'_0 s \\ b_r &= \left(1 - \frac{Q}{Q_0} t'_0 \right) b_{in} - i \frac{Q}{Q_0} \sqrt{\frac{\Gamma}{2}} t'_0 s.\end{aligned}\tag{C4}$$

We obtain the stationary solution for s

$$s = -i \sqrt{\frac{2}{\Gamma}} \frac{2s_z b_{in} t'_0}{t'_0 + \frac{1}{f} + \frac{2i\Delta\omega}{\Gamma} \frac{Q_0}{Q}}.\tag{C5}$$

Injecting this solution in the evolution equation for s_z , we find

$$\dot{s}_z = 0 = \frac{1}{2} + s_z \left(1 + \frac{|b_{in}|^2}{P'_c} \right),\tag{C6}$$

with

$$\frac{1}{P'_c} = \frac{2|t'_0|^2}{\Gamma(\Re(t'_0) + 1/f)} \left(\frac{1}{1/f + 2i\frac{Q_0}{Q}\frac{\Delta\omega}{\Gamma} + t'_0} + cc \right).\tag{C7}$$

Noting that $2\Re(t'_0) + 1/f = 2/f - t'_0 - t'^{*}_0$ we have

$$P'_c = \frac{\Gamma}{4|t'_0|^2} \left(\frac{1}{f^2} + \frac{1}{f} (t'_0 + t'^{*}_0) + \frac{2i\Delta\omega}{\Gamma} \frac{Q_0}{Q} (-t'_0 + t'^{*}_0) + \left(\frac{Q_0}{Q} \frac{2\Delta\omega}{\Gamma} \right)^2 + |t'_0|^2 \right)\tag{C8}$$

Let's remind here of the following expressions

$$\begin{aligned}\frac{1}{|t'_0|^2} &= \left(\frac{Q}{Q_0} \right)^2 + \left(\frac{\Delta\omega + \delta}{\kappa} \right)^2 \\ \frac{t'_0 + t'^{*}_0}{|t'_0|^2} &= 2 \\ \frac{-t'_0 + t'^{*}_0}{|t'_0|^2} &= \frac{2iQ}{Q_0} \frac{\Delta\omega + \delta}{\kappa}\end{aligned}\tag{C9}$$

We finally obtain

$$P'_c = \frac{\Gamma}{4} \phi'(\Delta\omega) , \quad (\text{C10})$$

with

$$\phi'(\omega) = \left(1 + \frac{1}{f}\right)^2 + \left(\frac{Q}{Q_0 f} \frac{\Delta\omega + \delta}{\kappa}\right)^2 + \left(\frac{2\Delta\omega}{\Gamma} \frac{Q_0}{Q}\right)^2 + \left(\frac{2\Delta\omega}{\Gamma} \frac{\Delta\omega + \delta}{\kappa}\right)^2 - \left(\frac{4\Delta\omega}{\Gamma} \frac{\Delta\omega + \delta}{\kappa}\right) . \quad (\text{C11})$$

If the system has no leaks, we have

$$\phi'(\Delta\omega) = \phi(\Delta\omega) = \left(\frac{2\Delta\omega}{\Gamma}\right)^2 + \left(\frac{2\Delta\omega}{\Gamma} \frac{\Delta\omega + \delta}{\kappa} - 1\right)^2 \quad (\text{C12})$$

Whatever the driving frequency may be, the absorption cross section remains positive. This expression is mainly used in section III. At resonance, we find

$$\phi'(0) = \frac{1}{\xi} = \left(1 + \frac{1}{f}\right)^2 , \quad (\text{C13})$$

which was exploited in section IV.

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